Formative Conversation Starters: Math

GRADE 5

nwea Professional Learning

Table of Contents

Formative Conversation Starters 3

5.1 Place Value.. 6

5.2 Equations and Place Value 12

5.3 Fractions.. 17

5.4 Fractions 24

5.5 Measurement and Formulas 29

Formative Conversation Starters

Student understanding is more about growing than it is about getting. As educators, we might speak of some students "getting it" and other students not. This conclusion, though, is not fair to students. A student who may not seem to "get it" does understand *something*, even if that *something* may not yet have grown into a robust web of thinking. To reach students' full knowledge, we can use guided conversations: Formative Conversation Starters.

Purpose: The purpose of Formative Conversation Starters is to help teachers reveal student understanding about key ideas in mathematics and to identify their students' ways of thinking.

Audience: The intended audience is teachers, who will use these questioning strategies with students.

Application: Teachers may wish to use these conversation starters in one-on-one conferences with students or in small groups.

Formative Conversation Starters approach student knowledge by presenting a single standards-based assessment item and leveraging the item to elicit conversation through clustered questioning. The goal of this activity is not to tell students what to think, but to help teachers better uncover how students are currently thinking about mathematical concepts. The conversations provide opportunities for students to communicate how they are thinking about mathematics.

Mathematical Ideas (BINSS – Big Ideas to Nurture Standards Sense-making)

As you read through these items, you will notice that we draw attention to a few specific mathematical ideas. These ideas correspond to important ways of thinking that all students should develop and continue to refine. They include:

- **Operations:** Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

- **Place Value:** Knowledge of place value is essential, and students should develop ways of thinking about place value that enable them to see the relationships between places. For example, they can think of a value in one place as 10 times that same value in the place to the right (*or a bundle of 10*), and they can carry that thinking between any places in any direction. They should be able to use that understanding effortlessly to compose and decompose quantities and to connect place-value understanding to operations.

- **Comparisons:** Comparisons can be either additive or multiplicative, with context guiding which is most appropriate. A multiplicative comparison is relative, describing one quantity in terms of another (e.g., 6 meters is <u>3 times as large</u> as 2 meters). Additive comparisons are absolute; the comparison is based on some other quantity (e.g., 6 meters is <u>4 meters more</u> than 2 meters). Students should have ways of thinking that help them determine which comparison to use or how an existing comparison is additive or multiplicative.

- **Measurement:** Geometric measurement is ultimately understood as the result of a multiplicative comparison between common attributes of two measurable quantities, and the result describes how many copies of a are contained in b. Equivalently, measurement addresses a times-as-large comparison such as "a is n times as large as b." Students thinking about measurement should have a clear understanding of which attribute is being measured and the comparison of two objects with that attribute, where one object's attribute is measured in terms of the other.

- **Fractions:** A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

- **Formulas:** Mastery of formulas (and procedures) is not the goal. Formulas are not the mathematics; they should be seen as shortcuts to help accomplish something with the mathematics. Students should have ways of thinking about the formulas that enable them to make sense of the quantities and to determine why quantities are connected with the indicated operations. Students should have mental and mathematical ways to reinvent useful formulas (e.g., $A=\pi r^2$ means three-and-a-bit copies of the square with area r^2).

- **Variables:** Students should have ways of thinking that enable them to distinguish between unknowns that vary and unknowns that represent some fixed value. For example, in the equation $6=3x+2$, x represents some unknown fixed value that makes the equation true. In $y=3x+2$, y and x vary with each other. In $y=mx+b$, y and x vary with each other, while m and b are typically nonvarying constants (parameters) within a problem.

- **Covarying Quantities:** Single quantities can vary, but students also need to consider situations where two quantities vary together. For example, in the equation $y=3x+2$, as the quantity x varies, the quantity y varies. It can be helpful to think about the relationship between covarying quantities in terms of how changes in x result in changes in y (e.g., as x increases by 1, y changes by . . .).

- **Proportional Relationships:** Proportional relationships require two covarying quantities. Those quantities must be measurable in some way, and the measures of those quantities scale in tandem. When one quantity changes by a scale factor, the other quantity also changes by the same scale factor. For example, doubling one quantity's value results in a doubling of the other quantity's value. Students should have ways of thinking that allow them to distinguish the two varying quantities in any proportional relationship and to explain how the quantities change by the same scale factor. It is important to note that proportional relationships are not synonymous with proportions. See this whitepaper for more information.

- **The Equal Sign:** The equal sign works in multiple ways in mathematics, even though the symbol does not change. Students should be able to think about the symbol as being relational (e.g., 2+4=5+1) and as being operational (e.g., the output of a computation), and should be able to determine which role the symbol is playing based on the situation. Keep an eye out for when students may put together equal signs. If this happens, prompting the student with questions about what the equals sign means may help. For example, suppose a student writes: 9 x 8 = 72 = 126 – 72 = 54 (*in this case, the student might be using the equal sign to say "next, I will..."*).

In the Formative Conversation Starters, you will notice that we call out mathematical ideas and the ways of thinking associated with them when it makes sense to do so. We also group discussion prompts to focus the conversations. Some groupings target core understandings that underpin the content. Other groupings elicit flexibility of thinking or extend beyond the assessment item being discussed. All of these methods are intended to provide opportunities for the teacher to listen to students and to reflect on how students might be thinking about mathematics in the standards-based assessment item.

The progressive question-and-answer strategy can be used to elicit evidence of students' ways of thinking about a topic or concept, with the purpose of guiding instruction.

How to Conduct a Formative Conversation

1. The questions for each item were developed to help teachers elicit information about students' ways of thinking about the content in the item and about mathematical ideas. These questions are suggestions, however, and not intended to be used as a script. The conversations teachers have will vary by student. While the questions for an item are laid out in a progression, teachers should vary the order to adapt to students' responses. Teachers should also keep in mind that students' responses may point to ways of thinking that are not addressed by the provided questions. In these cases, teachers should pursue those student understandings with their own line of questioning. There are several actions teachers can take to prepare for formative conversations:

   a. Become very familiar with the task and the questions ahead of time. This ensures that teachers can select the most appropriate next question based on how the students are responding.

   b. Provide students with tools to help them answer the questions. Depending on the task, these tools might include a manipulative, drawing paper, graph paper, or individual whiteboards and markers.

c. Have a list of questions to help further probe what students are thinking. Some examples are:

   i. Can you tell me more about that?

   ii. You look like you're really thinking about this. What are you thinking?

   iii. Can you draw me a picture/write an equation?

   iv. How did you get that answer?

   v. Is there another way that you could find that answer?

d. Make a plan to track what students say during the conversation: record the conversation, take notes, or have an observer take notes.

2. Teachers should ask questions without judgment. Student responses should not be labeled as right or wrong, and follow-up questions should be asked regardless of whether students give a correct response. Teachers should avoid commenting on students' responses other than to ask follow-up questions or to clarify what a student has said. Other students, however, should be encouraged to agree or disagree in a small group setting.

3. One of the most important parts of the formative conversation is what comes after the conversation: how will a teacher use the information about student thinking when planning instruction. Consider these suggestions for how to act on a formative conversation:

   a. Identify the different mathematical ideas addressed in the formative conversation. Where did students make connections between the ideas? Where do the connections need to be strengthened?

   b. Identify what students already understand in order to build instruction on that understanding.

   c. Identify areas where students can deepen what they already understand.

   d. Identify ways that students are comfortable with expressing mathematical ideas, and plan how to expand their capabilities. How were students most comfortable expressing or explaining what they understand? Were they more inclined to create a graphical representation of their thinking? Did they prefer to explain verbally? Did they use equations, or did they prefer to use graphs?

4. Some Conversation Starters include extension questions. These questions are provided as a way to elicit thinking beyond the grade-level of the BINSS.

5.1 Place Value

This activity focuses on student thinking about place value and base-ten understanding.

Place Value: Knowledge of place value is essential, and students should develop ways of thinking about place value that enable them to see the relationships between places. For example, they can think of a value in one place as 10 times that same value in the place to the right (or a *bundle of 10*), and they can carry that thinking between any places in any direction. They should be able to use that understanding effortlessly to compose and decompose quantities and to connect place-value understanding to operations.

ITEM ALIGNMENT

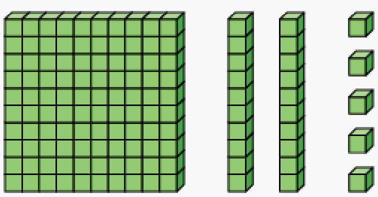
CCSS: 5.NBT.A.1

This item focuses on place value and base-ten understanding. However, it also provides an opportunity to talk about fractions, decimals, and area.

THE CONVERSATION STARTER

Use the information to answer the question.

The number represented by the model is different depending on which block represents 1.



Based on which block represents 1, what number does the model represent? Enter the answers in the boxes.

Smallest block represents 1:

Largest block represents 1:

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Content: Place Value (Meaning)

How does place value work?

- How many tens are in the number 125,100?
  - How many tens are in 100?
  - How many tens are in 5,000? 5,100?
  - How many tens are in 25,000?
- Suppose someone tells you that 403 = 43 because zeros have no value. What do you say?
- How does the value of 3 differ in the numbers 4.03 and 4.3? How could you compare the value of the 3s in both numbers?

B. Content: Place Value (Representation)

For our discussion today, we will refer to the “smallest block” as a single block, the “stack of 10 blocks” as a rod and the “largest block” as a flat. In the problem, how can the same block represent different values?

- If a single block represents 1, then what is the relationship between a single block and a rod?
- If the largest block (the flat) represents 1, then what is the relationship between a single block and a rod?
- Does this relationship change when you change which block represents 1? Why or why not?

C. Content: Place Value (Representation, Importance of the Whole)

What does the model represent when the single block represents 1?

- What does the model represent when the flat represents 1?

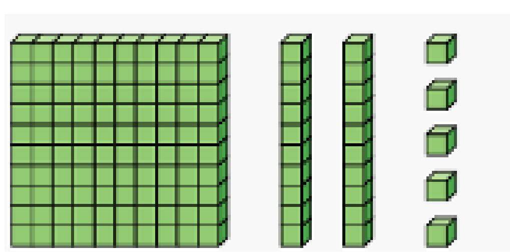
What is the relationship between what the model represents when the single block has a value of 1 and what it represents when the flat has a value of 1?

If the flat represents 10, what number does the model represent? How does this compare to when the single block represents 1?

What other numbers could the model represent? What would the value of each block for each of these numbers be?

D. Content: Fractions (Representation, Meaning)

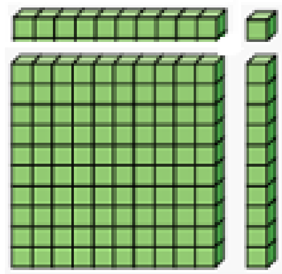
What fraction does the model represent when the flat represents 1? How do you know?



Could this model represent $\frac{125}{1000}$? If so, how?

E. Content: Fractions (Representation)

Look at the blocks and tell me when you see how this model shows 11 × 11.



- Tell me when you see how the model shows 1.1 × 1.1.

CONVERSATION PATHS (ANNOTATED)*

A. Content: Place Value (Meaning)

How does place value work?

Each place has 10 times the value of the place to its right.

- How many tens are in the number 125,100?

  Continue if students answer "none" or "0."

  - How many tens are in 100?

    10

  - How many tens are in 5,000? 5,100?

    500, 510

  - How many tens are in 25,000?

    2,500

- Suppose someone tells you that 403 = 43 because zeros have no value. What do you say?

  The value of a digit in a number is based on its place in the number. In both 403 and 43 there are 3 ones. The zero in 403 looks like there are no tens, but actually there are 40 tens; the number 43 has 4 tens. 403 can also be thought of as having four hundreds (which is equivalent to 40 tens), zero tens (aside from the 40 tens that are included in the 4 hundreds), and three ones.

- How does the value of 3 differ in the numbers 4.03 and 4.3? How could you compare the value of the 3s in both numbers?

  The 3 in 4.03 represents 3 hundredths. The 3 in 4.3 represents 3 tenths. The 3 in 4.03 is $\frac{1}{10}$ the value of the 3 in 4.3. The 3 in 4.3 is 10 times the value of the 3 in 4.03.

B. Content: Place Value (Representation)

For our discussion today, we will refer to the "smallest block" as a single block, the "stack of 10 blocks" as a rod and the "largest block" as a flat. In the problem, how can the same block represent different values?

The value that the block represents depends upon what is defined as one whole.

- If a single block represents 1, then what is the relationship between a single block and a rod?

  The rod has a value 10 times the value of the single block. The single block has a value $\frac{1}{10}$ the value of the rod.

- If the largest block (the flat) represents 1, then what is the relationship between a single block and a rod?

  The rod has a value 10 times the value of the single block. The single block has a value $\frac{1}{10}$ the value of the rod.

- Does this relationship change when you change which block represents 1? Why or why not?

  The relationship stays the same no matter which block represents 1. The rod will always have a value 10 times the value of a single block because it is composed of 10 single blocks, and the single block will always have a value $\frac{1}{10}$ the value of the rod because it is $\frac{1}{10}$ the size of the rod.

C. Content: Place Value (Representation, Importance of the Whole)

What does the model represent when the single block represents 1?

When the single block represents 1, the model represents 125.

- What does the model represent when the flat represents 1?

  When the flat represents 1, the model represents 1.25 or 1¼.

What is the relationship between what the model represents when the single block has a value of 1 and what it represents when the flat has a value of 1?

The number the model represents when the single block has a value of 1 is 100 times the number it represents when the flat has a value of 1.

If the flat represents 10, what number does the model represent? How does this compare to when the single block represents 1?

When the flat represents 1 ten, each column in it represents 1 and each cube represents 0.1, so the value of the model is 12.5. This is ⅒ the value of when the smallest block represented 1.

What other numbers could the model represent? What would the value of each block for each of these numbers be?

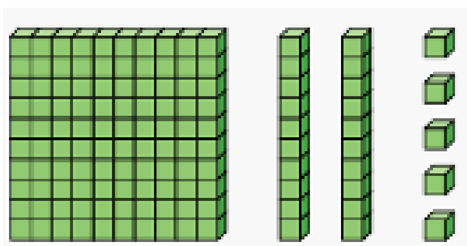
For example: The model could represent 0.125 if the flat represents 0.1, each rod represents 0.01, and each block represents 0.001; it could represent 125,000 if the flat represents 100,000, each rod represents 10,000, and each block represents 1,000.

D. Content: Fractions (Representation, Meaning)

What fraction does the model represent when the flat represents 1? How do you know?

125⁄100. Since the flat represents 1 and is partitioned into 100 cubes, the denominator is 100. The two rods are each ⅒ of the flat and the cubes are each 1⁄100 of the flat.

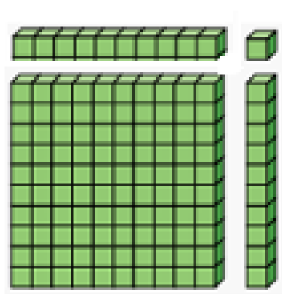
Could this model represent 125⁄1000? If so, how?



Yes, if the value of the flat is ⅒.

E. Content: Fractions (Representation)

Look at the blocks and tell me when you see how this model shows 11 × 11.



The flat is partitioned into 10 rows of 10 and 10 columns of 10. When you include the rod and the cube along the top and the side, you get 11 rows of 11 or 11 columns of 11. So, this is an area model for 11 × 11 = 121.

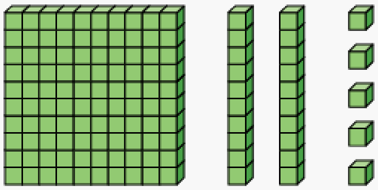
- Tell me when you see how the model shows 1.1 × 1.1.

  The flat represents 1 and each rod represents ⅒, or 0.1. The small block represents 1⁄100, or 0.01. So, 1.1 × 1.1 = 1.21.

5.1 Shareables*

Use the information to answer the question.

The number represented by the model is different depending on which block represents 1.

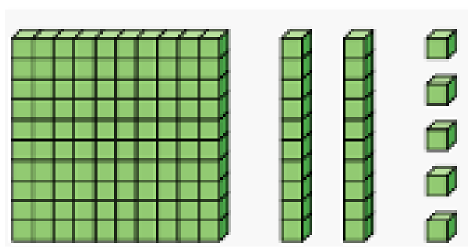


Based on which block represents 1, what number does the model represent? Enter the answers in the boxes.

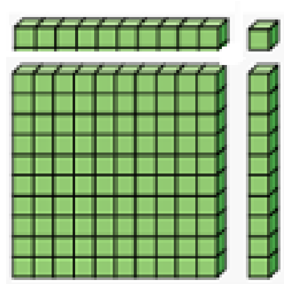
Smallest block represents 1:

Largest block represents 1:

D.



E.



5.2 Equations and Place Value

This activity focuses on student thinking about decimal computation.

Equations: A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

Place Value: Knowledge of place value is essential, and students should develop ways of thinking about place value that enable them to see the relationships between places. For example, they can think of a value in one place as 10 times that same value in the place to the right (or a *bundle of 10*), and they can carry that thinking between any places in any direction. They should be able to use that understanding effortlessly to compose and decompose quantities and to connect place-value understanding to operations.

ITEM ALIGNMENT	
CCSS: 5.NBT.B.7	***This item focuses on computing with decimals. However, it also provides an opportunity to talk about equations, operations, and relationships between operations and numbers.***

THE CONVERSATION STARTER

Use the equation to answer the question.

$\_\_\_\_\_ + 5.7 = 8.03$

What value makes the equation true? Enter the answer in the box.

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

How would you read the equation in this problem?

- What is this problem asking you to find?
- Is this a comparison question? What is being compared?

B. Problem Solving: Strategy

What are some ways that you could solve this problem?

- What is a real-life situation that could be described by this equation?
- How could you rewrite the equation using an operation that is not addition?

C. Content: Equations (Multiplication, Comparison)

Let's consider a similar equation: ____ × 5.7 = 8.03. Could this be thought of as a comparison question? Explain.

- How could you solve the problem ____ × 5.7 = 8.03?

D. Content: Operations (Division)

Using the equation 804 ÷ 3 = 268, and without calculating, can you tell me the value of each of the following expressions?

- 8.04 ÷ 3
- 80.4 ÷ 3
- 804 ÷ 0.3

How are these expressions alike? How are they different?

$$57 \times 803 \qquad 5.7 \times 8.03$$

Without calculating, how many times as large will the product of 57 × 803 be compared to the product of 5.7 × 8.03?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

How would you read the equation in this problem?

A number plus 5 and 7-tenths is equal to 8 and 3-hundredths. Or, the sum of a number and 5.7 is 8.03.

- What is this problem asking you to find?

  How much more than 5.7 the number 8.03 is.

- Is this a comparison question? What is being compared?

  Yes, you are comparing 5.7 to 8.03 to see how much more 8.03 is than 5.7.

B. Problem Solving: Strategy

What are some ways that you could solve this problem?

There are many possibilities. You could count up from 5.7 to 8.03. You could model both 5.7 and 8.03 with base-10 blocks and then compare the models to find the difference. You could model 8.03 with base-10 blocks and then remove 5.7 and identify the number that remains. You could subtract 5.7 from 8.03.

- What is a real-life situation that could be described by this equation?

  If students suggest a money context, encourage them to come up with a second context. Example contexts: A sandwich and a drink cost $8.03. The sandwich costs $5.70. How much does the drink cost? Or, one centipede measures 8.03 cm and another measures 5.7 cm. How much longer is the first centipede than the second centipede?

- How could you rewrite the equation using an operation that is not addition?

  You could rewrite this as a subtraction equation because addition and subtraction are related, inverse operations. You could rewrite this as 8.03 – 5.7 = ___ or as 8.03 – ___ = 5.7.

C. Content: Equations (Multiplication, Comparison)

Let's consider a similar equation: ___ × 5.7 = 8.03. Could this be thought of as a comparison question? Explain.

Yes. The blank represents how many copies of 5.7 are in 8.03.

- How could you solve the problem ___ × 5.7 = 8.03?

  Since it is a comparison, we can divide 8.03 by 5.7.

D. Content: Operations (Division)

Using the equation 804 ÷ 3 = 268, and without calculating, can you tell me the value of each of the following expressions?

- 8.04 ÷ 3

  2.68; 8.04 is 1⁄100 times as large as 804, so the result will be 1⁄100 times as large as 268, or 2.68.

- 80.4 ÷ 3

  26.8; 80.4 is 1⁄10 times as large as 804, so the result will be 1⁄10 times as large as 268, or 26.8.

- 804 ÷ 0.3

  2,680; since 0.3 is 1⁄10 times as large as 3, it will take 10 times as many copies of 0.3 (compared to 3) to make 804, so the result will be 2,680.

How are these expressions alike? How are they different?

$$57 \times 803 \qquad 5.7 \times 8.03$$

The values of each will contain the same digits in the same order. The second expression will have a value that is $\frac{1}{1000}$ that of the first expression, because we multiply numbers in the second expression that are $\frac{1}{10}$ and $\frac{1}{100}$ the values in the first expression.

Without calculating, how many times as large will the product of 57 × 803 be compared to the product of 5.7 × 8.03?

It will be 1,000 times as great because 57 is 10 times as large as 5.7 and 803 is 100 times as large as 8.03. 10 times 100 is 100.

5.2 Shareables*

Use the equation to answer the question.

$\_\_\_\_\_ + 5.7 = 8.03$

What value makes the equation true? Enter the answer in the box.

[ ]

D.

$$57 \times 803 \qquad 5.7 \times 8.03$$

5.3 Fractions

This activity focuses on student thinking about multiplying fractions.

Fractions: A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

ITEM ALIGNMENT

CCSS: 5.NF.B.4

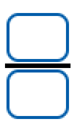
This item focuses on multiplication of fractions. However, it also provides an opportunity to talk about the meaning of operations and the meaning of fractions.

THE CONVERSATION STARTER

Multiply.

$\frac{6}{5} \times \frac{1}{2}$

Move numbers to the boxes to show the answer.



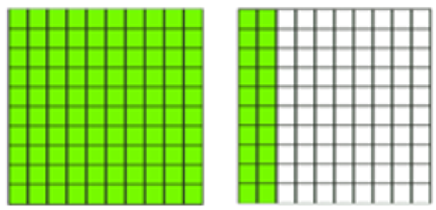
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Content: Fractions (Meaning)

What can you tell me about the fraction *a/b*?

- Is $\frac{6}{5}$ one or two numbers?
- What do you think of when you see the fraction $\frac{6}{5}$? What do the numbers in the fraction mean and how do they relate to one another?
- How are $\frac{6}{5}$ and $\frac{1}{2}$ alike and how are they different?
- Look at this diagram. Tell me when you see how it shows $\frac{6}{5}$.



- What are some other ways to write $\frac{6}{5}$?

B. Content: Multiplication (Meaning)

What does it mean to multiply?

What does 6 x 2 mean? What situation could it describe? How could you model it?

C. Content: Fractions (Multiplication)

Look at the diagram. Tell me when you see 6 one-halves. Tell me when you see half of 6. What situations could this model describe?

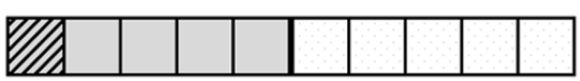
- Tell me when you see $6 \times \frac{1}{2}$ as $6 \div 2$ in the diagram.

D. Content: Fractions (Multiplication, Meaning)

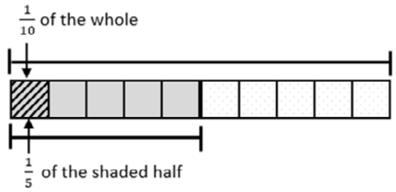
What does $\frac{5}{6} \times \frac{1}{2}$ mean?

- What situation could $\frac{5}{6} \times \frac{1}{2}$ describe?
- How could you model it?
- What does $\frac{6}{5} \times \frac{1}{2}$ mean?
  - What situation could it describe?
  - How could you model it?

Tell me when you see how $\frac{1}{5}$ and $\frac{1}{10}$ could describe the same region in this model. Explain how this makes sense.



- What equation could you write to relate ⅕ and ⅒ in this diagram?



- If two numbers are multiplied, when will the product be greater than either factor? When will the product not be greater than both factors? Explain using an example.

- Without calculating, which has the greater product: $\frac{6}{5} \times \frac{1}{2}$ or $\frac{5}{6} \times \frac{1}{2}$? How do you know?

$$\frac{6}{5} \text{ x } \frac{1}{2} \text{ or } \frac{5}{6} \text{ x } \frac{1}{2}$$

CONVERSATION PATHS (ANNOTATED)*

A. Content: Fraction (Meaning)

What can you tell me about the fraction *a/b*?

It is a single number that has a place on the number line. We can find that place by imagining "b" partitions between zero and 1, each with a value of 1/b, then copying that "a" times.

- Is $\frac{6}{5}$ one or two numbers?

  A fraction looks like two numbers, but it is a single number.

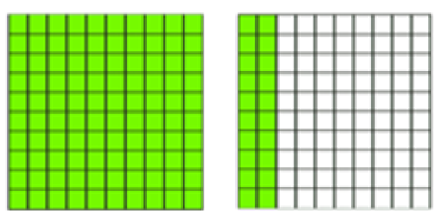
- What do you think of when you see the fraction $\frac{6}{5}$? What do the numbers in the fraction mean and how do they relate to one another?

  Do students understand $\frac{6}{5}$ as 6 parts of a segment divided into 5 equal parts? Do they understand it as a number greater than 1? Do they understand the relationship to the unit fraction—that $\frac{6}{5}$ is six copies of one-fifth? Do they see 6 copies of $\frac{1}{5}$ where $\frac{1}{5}$ is created by cutting a whole into 5 equal parts and each part is $\frac{1}{5}$ of the whole? Do they see it as 6 divided by 5 or as a comparison of 6 to 5?

- How are $\frac{6}{5}$ and $\frac{1}{2}$ alike and how are they different?

  In both, we think about partitioning a whole (such as a segment or shape) into a given number of equal pieces (5 pieces in the case of $\frac{6}{5}$ and 2 pieces in the case of $\frac{1}{2}$). Since $\frac{6}{5}$ is 6 copies of $\frac{1}{5}$ and since 5 copies of $\frac{1}{5}$ is a whole unit, we know $\frac{6}{5}$ is greater than 1. However, $\frac{1}{2}$ is less than 1. Some might call $\frac{6}{5}$ an improper fraction, but it is as valid as a mixed number. We might call $\frac{1}{2}$ a unit fraction.

- Look at this diagram. Tell me when you see how it shows $\frac{6}{5}$.



  If the flat represents a whole, then two rods would represent $\frac{1}{5}$, and there are 6 pairs of two rods.

- What are some other ways to write $\frac{6}{5}$?

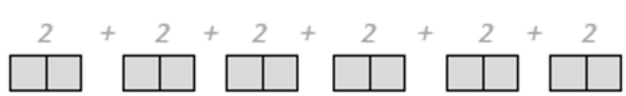
  $1\frac{1}{5}$ or $\frac{12}{10}$ or 1.2

B Content: Multiplication (Meaning)

What does it mean to multiply?

Do students see multiplication only as combining equal groups, or do they also understand it as comparison or scaling? In this case, $\frac{6}{5}$ x $\frac{1}{2}$ means that we are computing $\frac{6}{5}$ copies of $\frac{1}{2}$ or a value that is $\frac{6}{5}$ times as large as $\frac{1}{2}$.

What does 6 x 2 mean? What situation could it describe? How could you model it?



It might mean combining equal groups of the same thing: 6 bags with 2 muffins in each bag. Alternatively, it means 6 copies of 2 or 6 times as large as 2. If we are computing 6 copies of 2, we are thinking about 2 + 2 + 2 + 2 + 2 + 2 = 12.

It can also mean comparing different groups multiplicatively: 6 times as many blueberry muffins as 2 corn muffins.

C Content: Fraction (Multiplication)

Look at the diagram. Tell me when you see 6 one-halves. Tell me when you see half of 6. What situations could this model describe?

For example, if each set of 2 blocks represents a whole, then one colored block in each set is $\frac{1}{2}$. There are 6 one-halves. For example, if each set of 2 blocks represents a whole, and there are 6 sets, then the total is 6 or 6 wholes. Half of 6 is represented by the 6 colored blocks. Shaina ran $\frac{1}{2}$ mile every day for 6 days. How many miles did she run in all? Or Shaina ran $\frac{1}{2}$ of a 6-mile trail.

- Tell me when you see $6 \times \frac{1}{2}$ as $6 \div 2$ in the diagram.

Each colored block represents $\frac{1}{2}$ and there are 6 of them. At the same time, 6 is divided by 2 because each whole is divided by 2.

D. Content: Fraction (Multiplication, Meaning)

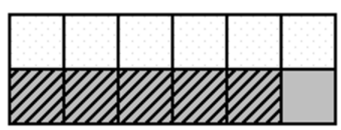
What does $\frac{5}{6} \times \frac{1}{2}$ mean?

It means $\frac{5}{6}$ of a copy of $\frac{1}{2}$. That is, not a full copy of $\frac{1}{2}$ but $\frac{5}{6}$ of a copy of $\frac{1}{2}$. Note that since $\frac{1}{2}$ is referencing a whole unit, $\frac{5}{6}$ of a copy of $\frac{1}{2}$ also references the original whole unit.

- What situation could $\frac{5}{6} \times \frac{1}{2}$ describe?

Hector had $\frac{1}{2}$ gallon of milk. He used $\frac{5}{6}$ of the milk in a recipe. How many gallons of milk did Hector use in the recipe?

- How could you model it?



For example, start with a rectangle to represent the whole unit, then add a horizontal line to cut the figure in half. The top portion is $\frac{1}{2}$ of the whole. Next, add vertical lines to partition the original rectangle into 6 equal parts, creating 12 small regions that each now represent $\frac{1}{6}$ of the $\frac{1}{2}$ of the whole. We need 5 copies of this $\frac{1}{6}$ of the $\frac{1}{2}$ of the whole. When referencing the original whole again, we see it as $\frac{5}{12}$ of the whole.

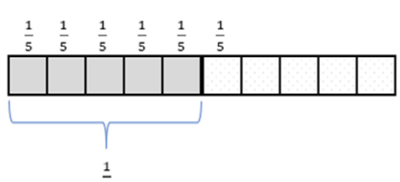
- What does $\frac{6}{5} \times \frac{1}{2}$ mean?

  $\frac{6}{5}$ of a copy of $\frac{1}{2}$. That is, more than a full copy of $\frac{1}{2}$. Note that since $\frac{1}{2}$ is referencing a whole unit, $\frac{6}{5}$ of a copy of $\frac{1}{2}$ also references the original whole unit.

- What situation could it describe?

  Shaina ran $\frac{1}{2}$ mile. Lindsay ran $\frac{6}{5}$ as far as Shain.

- How could you model it?



For example, start with a rectangle to represent the whole unit—which is 1 mile in this context. Cut the rectangle into two equal parts. Each part represents ½ mile. Next, cut each ½ mile section of the rectangle into 5 equal parts (each representing ⅕ of the ½ of the whole). We need 6 copies of these ⅕ of the ½ of the whole. When referencing the original whole again, we see it as $\frac{6}{10}$ of the whole. Therefore, Lindsay ran $\frac{6}{10}$ of a mile.

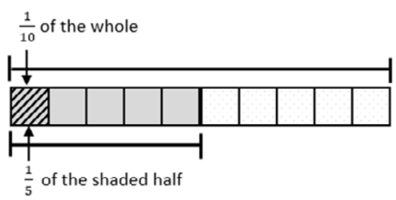
Tell me when you see how ⅕ and ⅒ could describe the same region in this model. Explain how this makes sense.

The ⅕ sections are based on cutting ½ of the rectangle into 5 equal sections. So, ⅕ references half of the rectangle. If each of the half rectangles is cut into 5 equal sections, the whole rectangle has 10 sections. The ⅒ references the whole rectangle. So, ⅕ of ½ is the same as ⅒ of the whole.

- What equation could you write to relate ⅕ and ⅒ in this diagram?

  Since ⅕ of ½ is the same as ⅒ of the whole, we have ⅕ x ½ = ⅒.

  Annotated diagram:



- If two numbers are multiplied, when will the product be greater than either factor? When will the product not be greater than both factors? Explain using an example.

  For positive numbers (the only kind mentioned up to grade 5), the product is only greater than both factors if each factor is greater than 1. If either factor is less than 1, then the product of those factors will not be greater than both factors. For example, $\frac{4}{3}$ x ½ means $\frac{4}{3}$ times as large as ½, which is less than one.

- Without calculating, which has the greater product: $\frac{6}{5}$ × ½ or ⅚ × ½? How do you know?

$$\frac{6}{5} \text{ x} \frac{1}{2} \text{ or } \frac{5}{6} \text{ x } \frac{1}{2}$$

The product of $\frac{6}{5}$ × ½ is greater because one factor, $\frac{6}{5}$, is greater than 1, whereas both ⅚ and ½ are less than 1, so their product will be smaller.

5.3 Shareables*

Multiply.

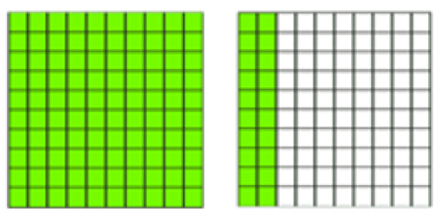
$\frac{6}{5} \times \frac{1}{2}$

Move numbers to the boxes to show the answer.

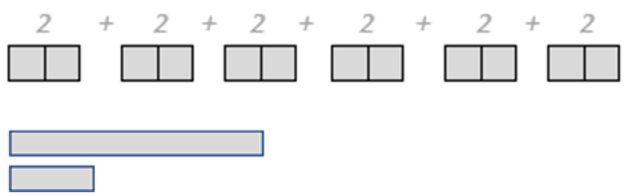
$\frac{\square}{\square}$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

A.

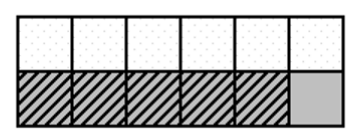


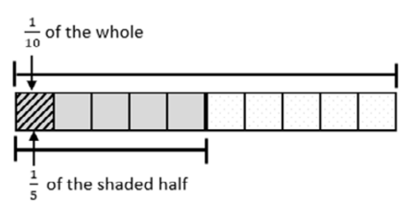
B.

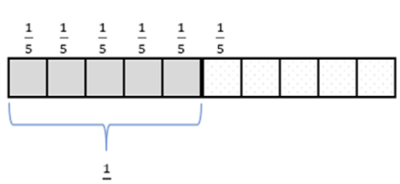


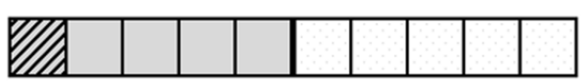
C.

D.









$$\frac{6}{5} \text{ x } \frac{1}{2} \text{ or } \frac{5}{6} \text{ x } \frac{1}{2}$$

5.4 Fractions

This activity focuses on student thinking about fraction computation.

Fractions: A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

ITEM ALIGNMENT

CCSS: 5.NF.B.7c

This item focuses on division with fractions and whole numbers. However, it also provides an opportunity to talk about the relationship between multiplication and division and understanding of fractions.

THE CONVERSATION STARTER

Use the information to answer the question.

Jade has $\frac{1}{2}$ yard of wire. She cuts the wire into 3 equal-length pieces for a science project.

What is the length of each piece of wire in yards? Move numbers into the boxes to show the answer. If there is no whole number, put a 0 in the first box.

0	1	2	3	4	5	6	7	8	9	
10	11	12	13	14	15	16	17	18	19	20

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

Can you explain this situation in your own words?

B. Problem Solving: Strategy

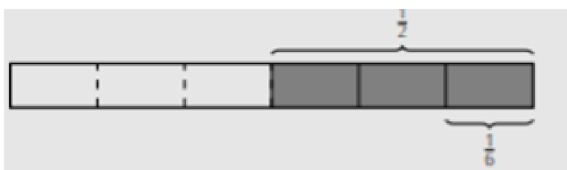
Tell me when you see this as a division problem.

- Tell me when you see this as a multiplication problem.
- How could you write an equation for this situation using division? Using multiplication?
- Without solving, will the answer be greater or less than ½ yard? How do you know?

C. Content: Fractions (Division)

Draw a diagram that represents 1 yard. How could you model this problem using this diagram?

- What fractional part of the whole does one of the three pieces represent?



D. Content: Fractions (Comparison)

Let's look at another problem. "Camilo has ½ yard of wire. He needs 3 yards of wire. What fraction of what he needs does Camilo have?" How is this problem different?

- How could you model Camilo's situation?
- How could you write this as an equation?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

Can you explain this situation in your own words?

A person has a length of wire that she wants to cut into equal-length pieces.

B. Problem Solving: Strategy

Tell me when you see this as a division problem.

It is a division problem because division is about finding either how many equal groups there are or how many are in each equal group. Here we are trying to find how long each segment (how many units in each group) will be if there are three equal-sized segments (three equal groups).

- Tell me when you see this as a multiplication problem.

  We can view it as a multiplication problem because the 3 equal groups combine to make ½ yard of wire.

- How could you write an equation for this situation using division? Using multiplication?

  ½ ÷ 3 = ☐ or 3 × ☐ = ½

- Without solving, will the answer be greater or less than ½ yard? How do you know?

  It will be less than ½ yard because you are cutting a fraction of a yard into even smaller pieces.

C. Content: Fractions (Division)

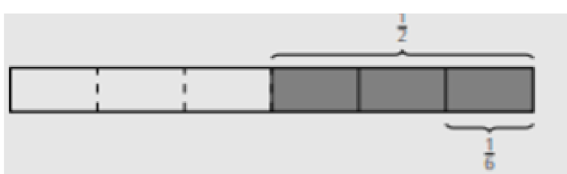
Draw a diagram that represents 1 yard. How could you model this problem using this diagram?

For example, start with a whole unit (the entire rectangle in this model) and cut it into two equal parts. Each of these is called ½. Next, we cut the ½ into 3 equal partitions as shown.

- What fractional part of the whole does one of the three pieces represent?

  Since the other half of the whole can also be divided into three equal parts, there will be a total of 6 equal pieces in 1 yard. One piece represents 1 out of 6 or ⅙ of a yard, or referencing the whole unit, ½ ÷ 3 = ⅙.

  Annotated diagram:



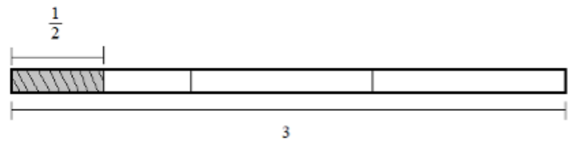
D. Content: Fractions (Comparison)

Let's look at another problem. "Camilo has ½ yard of wire. He needs 3 yards of wire. What fraction of what he needs does Camilo have?" How is this problem different?

In the first problem, we are trying to find the length of each segment when ½ yard of wire is cut into 3 equal parts. We know the total and the number of groups but not the amount or length of each group. In this problem, you know how much Camilo has and how much he needs. The relationship between the amounts is different. We can divide the whole number by the fraction.

- How could you model Camilo's situation?

  There are 3 yards total, representing the amount Camilo needs, and one of the yards is divided in half to represent the amount of wire Camilo has. For example:



- How could you write this as an equation?

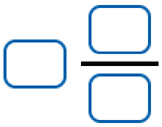
  3 ÷ ½ = ☐ is one possibility.

5.4 Shareables*

Use the information to answer the question.

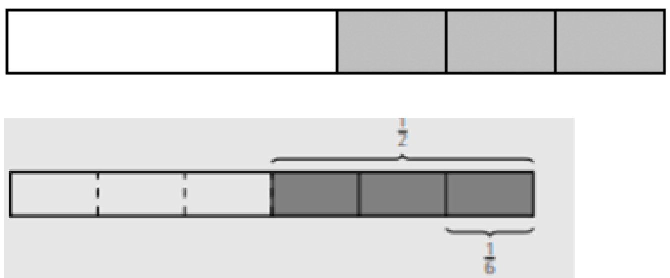
Jade has $\frac{1}{2}$ yard of wire. She cuts the wire into 3 equal-length pieces for a science project.

What is the length of each piece of wire in yards? Move numbers into the boxes to show the answer. If there is no whole number, put a 0 in the first box.



0	1	2	3	4	5	6	7	8	9	
10	11	12	13	14	15	16	17	18	19	20

C.



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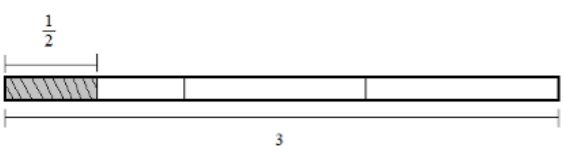


5.5 Measurement and Formulas

This activity focuses on student thinking about volume.

Measurement: Geometric measurement is ultimately understood as the result of a multiplicative comparison between common attributes of two measurable quantities, and the result describes how many copies of a are contained in b. Equivalently, measurement addresses a times-as-large comparison such as "a is n times as large as b." Students thinking about measurement should have a clear understanding of which attribute is being measured and the comparison of two objects with that attribute, where one object's attribute is measured in terms of the other.

Formulas: Mastery of formulas (and procedures) is not the goal. Formulas are not the mathematics; they should be seen as shortcuts to help accomplish something with the mathematics. Students should have ways of thinking about the formulas that enable them to make sense of the quantities and to determine why quantities are connected with the indicated operations. Students should have mental and mathematical ways to reinvent useful formulas (e.g., $A=\pi r^2$ means three-and-a-bit copies of the square with area r^2).

ITEM ALIGNMENT

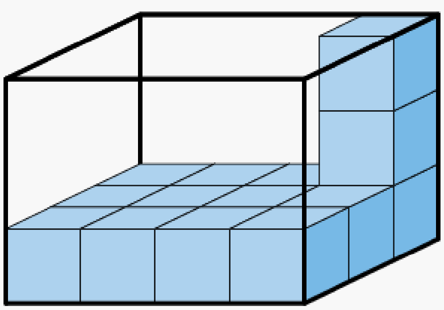
CCSS: 5.MD.C.4

This item focuses on volume. However, it also provides an opportunity to talk about the relationship between linear measure, area, and formulas, as well as unit conversions.

THE CONVERSATION STARTER

Use the information to answer the question.

Sam is filling a rectangular prism with unit cubes.



He has already placed 14 cubes in the rectangular prism.

How many total unit cubes will fit in the prism? Enter the answer in the box. Be sure to include the cubes that Sam has already placed.

[   ] unit cubes

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

What is this question asking you?

- What is volume?
- How is this question like a question about volume? How is asking about cubes or counting cubes like measuring volume?
- What is a unit cube?
- Why are cubes used to measure volume?
- What type of units are used to measure volume? Why?

B. Content: Volume

How can you figure out the number of cubes when not all of the cubes are shown?

- What are some ways you can measure the volume of this prism? What operations could you use to find the volume?
- Is there a way to figure out how many cubes are in the bottom layer without counting all of them?
- How do the number of cubes in the bottom layer compare to the volume?
- What is the length of the prism? What is the width? What is the height? How do you know?

C. Content: Unit Conversions

Imagine that the cubes in the situation are 1 foot long on each edge. Therefore, one cube represents 1 cubic foot, making the volume of the prism 36 cubic feet. How many cubic inches would this be?

- How many inches are there in 1 foot?
- How many square inches are there in a square foot?
- How many cubic inches are there in a cubic foot?

D. Content: Volume and Area (Formulas)

Let's think about area for a moment. Is area a one-dimensional, two-dimensional, or three-dimensional measure?

- Does area have thickness?
- A formula that can be used to find the volume of a prism: $V=Bh$, where B is the area of the base of the prism. How can that make sense when area is two-dimensional? Wouldn't h copies of a two-dimensional object still be two-dimensional?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

What is this question asking you?

The question is asking us to identify the total number of unit cubes that it will take to fill the prism. It is asking about the volume of the prism.

- What is volume?

  Volume is the three-dimensional space an object takes up. It's an attribute we can measure.

- How is this question like a question about volume? How is asking about cubes or counting cubes like measuring volume?

  This question is asking you to find how much space the prism takes up by comparing it, multiplicatively, to the volume of another figure—a unit cube.

- What is a unit cube?

  A unit cube is a 3-dimensional shape in which all of the sides are the length of 1 unit of some linear measure.

- Why are cubes used to measure volume?

  Since each cube has a volume of one cubic unit, each cube that you use to fill an object, ideally without gaps or overlaps, represents 1 cubic unit and the total number of cubes that fit without gaps represents the volume of the object being measured.

- What type of units are used to measure volume? Why?

  Volume can only be measured in comparison to some other volume. We often use cubes that fit well together and leave no gaps. Volume is commonly expressed in terms of the number of unit cubes that it takes to fill an object with no gaps.

B. Content: Volume

How can you figure out the number of cubes when not all of the cubes are shown?

We can see there are 12 cubes in the bottom layer and there are 3 layers. So, there will be 3 groups of 12 cubes, or 36 cubic units.

- What are some ways you can measure the volume of this prism? What operations could you use to find the volume?

  If we measure in terms of a multiplicative comparison to a given cube, we could add 12 + 12 + 12 or multiply 3 × 12 to find the volume of the prism is 36 times as large as one cube.

- Is there a way to figure out how many cubes are in the bottom layer without counting all of them?

  You can count how many cubes are along the front and how many are along the side and multiply to find the total number of cubes in one layer.

- How do the number of cubes in the bottom layer compare to the volume?

  The total volume is 3 times as great as the volume of the bottom layer, or base.

- What is the length of the prism? What is the width? What is the height? How do you know?

  We need to find something with the attribute of length to use to describe these measurements. The one that is most helpful is the length of one edge of a cube, so let's use that. Let's call that a "unit length." The prism is 4 "unit lengths" long, 3 "unit lengths" wide, and 3 "unit lengths" high.

C. Content: Unit conversions

Imagine that the cubes in the situation are 1 foot long on each edge. Therefore, one cube represents 1 cubic foot, making the volume of the prism 36 cubic feet. How many cubic inches would this be?

Since each cubic foot is 12 inches by 12 inches by 12 inches, or 1,728 cubic inches, we have 36 copies of 1,728 cubic inches, or 62,208 cubic inches.

- How many inches are there in one foot?

  12

- How many square inches are there in a square foot?

  144

- How many cubic inches are there in a cubic foot?

  1,728 cubic inches

D. Content: Volume and Area (Formulas)

Let's think about area for a moment. Is area a one-dimensional, two-dimensional, or three-dimensional measure?

It is two-dimensional.

- Does area have thickness?

  No, it is only two-dimensional. Think about a piece of paper, but not about its thickness. Area is two-dimensional, like a plane. Volume has "thickness" because it has a third dimension.

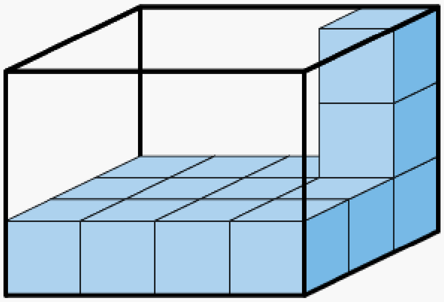
- A formula that can be used to find the volume of a prism: $V=Bh$, where B is the area of the base of the prism. How can that make sense when area is two-dimensional? Wouldn't h copies of a two-dimensional object still be two-dimensional?

  If we think of B not as the area of the base, but instead as counting the unit cubes on the base/bottom layer, it makes sense.

5.5 Shareables*

Use the information to answer the question.

Sam is filling a rectangular prism with unit cubes.



He has already placed 14 cubes in the rectangular prism.

How many total unit cubes will fit in the prism? Enter the answer in the box. Be sure to include the cubes that Sam has already placed.

[ ] unit cubes

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