Formative Conversation Starters: Math

GRADE 4

nwea Professional Learning

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Formative Conversation Starters

Student understanding is more about growing than it is about getting. As educators, we might speak of some students "getting it" and other students not. This conclusion, though, is not fair to students. A student who may not seem to "get it" does understand *something*, even if that *something* may not yet have grown into a robust web of thinking. To reach students' full knowledge, we can use guided conversations: Formative Conversation Starters.

Purpose: The purpose of Formative Conversation Starters is to help teachers reveal student understanding about key ideas in mathematics and to identify their students' ways of thinking.

Audience: The intended audience is teachers, who will use these questioning strategies with students.

Application: Teachers may wish to use these conversation starters in one-on-one conferences with students or in small groups.

Formative Conversation Starters approach student knowledge by presenting a single standards-based assessment item and leveraging the item to elicit conversation through clustered questioning. The goal of this activity is not to tell students what to think, but to help teachers better uncover how students are currently thinking about mathematical concepts. The conversations provide opportunities for students to communicate how they are thinking about mathematics.

Mathematical Ideas (BINSS – Big Ideas to Nurture Standards Sense-making)

As you read through these items, you will notice that we draw attention to a few specific mathematical ideas. These ideas correspond to important ways of thinking that all students should develop and continue to refine. They include:

- **Operations:** Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

- **Place Value:** Knowledge of place value is essential, and students should develop ways of thinking about place value that enable them to see the relationships between places. For example, they can think of a value in one place as 10 times that same value in the place to the right (*or a bundle of 10*), and they can carry that thinking between any places in any direction. They should be able to use that understanding effortlessly to compose and decompose quantities and to connect place-value understanding to operations.

- **Comparisons:** Comparisons can be either additive or multiplicative, with context guiding which is most appropriate. A multiplicative comparison is relative, describing one quantity in terms of another (e.g., 6 meters is <u>3 times as large</u> as 2 meters). Additive comparisons are absolute; the comparison is based on some other quantity (e.g., 6 meters is <u>4 meters more</u> than 2 meters). Students should have ways of thinking that help them determine which comparison to use or how an existing comparison is additive or multiplicative.

- **Measurement:** Geometric measurement is ultimately understood as the result of a multiplicative comparison between common attributes of two measurable quantities, and the result describes how many copies of a are contained in b. Equivalently, measurement addresses a times-as-large comparison such as "a is n times as large as b." Students thinking about measurement should have a clear understanding of which attribute is being measured and the comparison of two objects with that attribute, where one object's attribute is measured in terms of the other.

- **Fractions:** A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

- **Formulas:** Mastery of formulas (and procedures) is not the goal. Formulas are not the mathematics; they should be seen as shortcuts to help accomplish something with the mathematics. Students should have ways of thinking about the formulas that enable them to make sense of the quantities and to determine why quantities are connected with the indicated operations. Students should have mental and mathematical ways to reinvent useful formulas (e.g., $A=\pi r^2$ means three-and-a-bit copies of the square with area r^2).

- **Variables:** Students should have ways of thinking that enable them to distinguish between unknowns that vary and unknowns that represent some fixed value. For example, in the equation $6=3x+2$, x represents some unknown fixed value that makes the equation true. In $y=3x+2$, y and x vary with each other. In $y=mx+b$, y and x vary with each other, while m and b are typically nonvarying constants (parameters) within a problem.

- **Covarying Quantities:** Single quantities can vary, but students also need to consider situations where two quantities vary together. For example, in the equation $y=3x+2$, as the quantity x varies, the quantity y varies. It can be helpful to think about the relationship between covarying quantities in terms of how changes in x result in changes in y (e.g., as x increases by 1, y changes by . . .).

- **Proportional Relationships:** Proportional relationships require two covarying quantities. Those quantities must be measurable in some way, and the measures of those quantities scale in tandem. When one quantity changes by a scale factor, the other quantity also changes by the same scale factor. For example, doubling one quantity's value results in a doubling of the other quantity's value. Students should have ways of thinking that allow them to distinguish the two varying quantities in any proportional relationship and to explain how the quantities change by the same scale factor. It is important to note that proportional relationships are not synonymous with proportions. See this whitepaper for more information.

- **The Equal Sign:** The equal sign works in multiple ways in mathematics, even though the symbol does not change. Students should be able to think about the symbol as being relational (e.g., 2+4=5+1) and as being operational (e.g., the output of a computation), and should be able to determine which role the symbol is playing based on the situation. Keep an eye out for when students may put together equal signs. If this happens, prompting the student with questions about what the equals sign means may help. For example, suppose a student writes: 9 x 8 = 72 = 126 – 72 = 54 (*in this case, the student might be using the equal sign to say "next, I will…"*).

In the Formative Conversation Starters, you will notice that we call out mathematical ideas and the ways of thinking associated with them when it makes sense to do so. We also group discussion prompts to focus the conversations. Some groupings target core understandings that underpin the content. Other groupings elicit flexibility of thinking or extend beyond the assessment item being discussed. All of these methods are intended to provide opportunities for the teacher to listen to students and to reflect on how students might be thinking about mathematics in the standards-based assessment item.

The progressive question-and-answer strategy can be used to elicit evidence of students' ways of thinking about a topic or concept, with the purpose of guiding instruction.

How to Conduct a Formative Conversation

1. The questions for each item were developed to help teachers elicit information about students' ways of thinking about the content in the item and about mathematical ideas. These questions are suggestions, however, and not intended to be used as a script. The conversations teachers have will vary by student. While the questions for an item are laid out in a progression, teachers should vary the order to adapt to students' responses. Teachers should also keep in mind that students' responses may point to ways of thinking that are not addressed by the provided questions. In these cases, teachers should pursue those student understandings with their own line of questioning. There are several actions teachers can take to prepare for formative conversations:

    a. Become very familiar with the task and the questions ahead of time. This ensures that teachers can select the most appropriate next question based on how the students are responding.

    b. Provide students with tools to help them answer the questions. Depending on the task, these tools might include a manipulative, drawing paper, graph paper, or individual whiteboards and markers.

   c. Have a list of questions to help further probe what students are thinking. Some examples are:

      i. Can you tell me more about that?

      ii. You look like you're really thinking about this. What are you thinking?

      iii. Can you draw me a picture/write an equation?

      iv. How did you get that answer?

      v. Is there another way that you could find that answer?

   d. Make a plan to track what students say during the conversation: record the conversation, take notes, or have an observer take notes.

2. Teachers should ask questions without judgment. Student responses should not be labeled as right or wrong, and follow-up questions should be asked regardless of whether students give a correct response. Teachers should avoid commenting on students' responses other than to ask follow-up questions or to clarify what a student has said. Other students, however, should be encouraged to agree or disagree in a small group setting.

3. One of the most important parts of the formative conversation is what comes after the conversation: how will a teacher use the information about student thinking when planning instruction. Consider these suggestions for how to act on a formative conversation:

   a. Identify the different mathematical ideas addressed in the formative conversation. Where did students make connections between the ideas? Where do the connections need to be strengthened?

   b. Identify what students already understand in order to build instruction on that understanding.

   c. Identify areas where students can deepen what they already understand.

   d. Identify ways that students are comfortable with expressing mathematical ideas, and plan how to expand their capabilities. How were students most comfortable expressing or explaining what they understand? Were they more inclined to create a graphical representation of their thinking? Did they prefer to explain verbally? Did they use equations, or did they prefer to use graphs?

4. Some Conversation Starters include extension questions. These questions are provided as a way to elicit thinking beyond the grade-level of the BINSS.

4.1 Comparisons

This activity focuses on student thinking about multiplicative comparison.

Comparisons: Comparisons can be either additive or multiplicative, with context guiding which is most appropriate. A multiplicative comparison is relative, describing one quantity in terms of another (e.g., 6 meters is 3 times as large as 2 meters). Additive comparisons are absolute; the comparison is based on some other quantity (e.g., 6 meters is 4 meters more than 2 meters). Students should have ways of thinking that help them determine which comparison to use or how an existing comparison is additive or multiplicative.

ITEM ALIGNMENT

CCSS: 4.OA.A.2

This item focuses on multiplicative comparisons. However, it also provides an opportunity to talk about comparisons in general, operations, fractions, and variables.

THE CONVERSATION STARTER

Use the information to complete the task.

Carlos painted 18 pictures. He painted 3 times as many pictures as Luis painted.

Make an equation to show the number of pictures Luis painted. Move numbers and symbols to the lines. Use the blank box to represent the number of pictures Luis painted.

________ = ____

□ 3 18 + − × ÷

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

Can you explain this situation in your own words?

What do you know, and what are you trying to find?

In the original problem about the number of paintings Luis painted, what operation or operations could you use to solve?

How are the following statements the same? How are they different?

3 times a number is 729
3 times 729 is a number

B. Content: Comparison (Additive, Multiplicative)

What does "times as many" mean?

How is "3 times as many" different from "3 more than"?

This problem can be reworded as "Luis painted ______ as many pictures as Carlos." Does this make sense? How?

C. Content: Comparison (Additive, Multiplicative)

How many different ways can you compare two numbers?

Compare 12 to 3 in as many ways as you can.

Complete these statements:

- 12 is ______ times as large as 3.
- 3 is ______ times as large as 12.

Compare 12 to 8 in as many ways as you can.

- Compare 11 to 3 in as many ways as you can.

Look at these equations. Tell me when you see a comparison in the following equations. What is being compared?

$$13 - \square = 8$$

$$15 - 8 = \square$$

$$12 \times 42 = \square$$

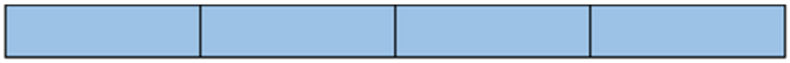
$$14 \div \square = 3$$

$$14 \div 5 = \square$$

D. Content: Fractions (Meaning)

If Luis painted ⅓ as many pictures as Carlos, what fraction of all the paintings were painted by Carlos?

How could you use this diagram to show that Luis painted ⅓ as many pictures as Carlos? What would each region represent?



How are these two statements alike? How are they different?

$\frac{1}{3}$ of a number is 18	$\frac{1}{3}$ of 18

E. Extension: Comparison (Multiplicative)

If Carlos always paints 3 times as many pictures as Luis, what will happen if the number of pictures Luis paints increases by 2?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

Can you explain this situation in your own words?

Carlos and Luis each painted some pictures. Carlos painted 18 pictures. This is 3 times the number of pictures Luis painted.

What do you know, and what are you trying to find?

We know the number of pictures Carlos painted. We also know that Carlos painted 3 times as many pictures as Luis. We are trying to find the number of pictures Luis painted.

In the original problem about the number of paintings Luis painted, what operation or operations could you use to solve?

Either multiplication or division can be used to think through the problem.

How are the following statements the same? How are they different?

3 times a number is 729
3 times 729 is a number

The following statements are similar in appearance, but very different. Can your students see the difference and make sense of each statement?

B. Content: Comparison (Additive, Multiplicative)

What does “times as many” mean?

It signals that the comparison between the quantities is multiplicative. We will need to focus on how one quantity will be some multiple of another (or perhaps on the number of copies of one quantity in another).

How is “3 times as many” different from “3 more than”?

The phrase “times as many” uses the other quantity as the point of reference, while the phrase “more than” uses a single object as the point of reference. The first is multiplicative, and the second is additive. The phrase “3 times as many” represents a quantity that is 3 times as much as another quantity.

This problem can be reworded as “Luis painted ______ as many pictures as Carlos.” Does this make sense? How?

Here, the relationship is flipped. How agile are your students in working with the reciprocal relationship?

C. Content: Comparison (Additive, Multiplicative)

How many different ways can you compare two numbers?

Additive or multiplicative comparison can be used. Also, “greater than” or “less than” can be used to compare numbers.

Compare 12 to 3 in as many ways as you can.

Additive comparison: 12 is 9 more than 3. Multiplicative comparison: 12 is 4 times as much as 3. Also, “greater than” language can be used: 12 is greater than 3.

Complete these statements:

- 12 is ______ times as large as 3.

  4

- 3 is ______ times as large as 12.

  ¼

Compare 12 to 8 in as many ways as you can.

> ***Additive comparison: 12 is 4 more than 8. Multiplicative comparison: 12 is 1.5 times as much as 3. Also, "greater than" language can be used: 12 is greater than 3.***

- Compare 11 to 3 in as many ways as you can.

  Additive comparison: 11 is 8 more than 3. Multiplicative comparison, 11 is 3 ⅔ times as many as 3.

Look at these equations. Tell me when you see a comparison in the following equations. What is being compared?

$$13 - \square = 8$$

$$15 - 8 = \square$$

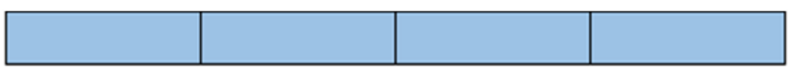
$$12 \times 42 = \square$$

$$14 \div \square = 3$$

$$14 \div 5 = \square$$

> ***For the first, one might say 13 is 8 more than the number in the box—an additive comparison. For the second, the unknown in the box represents how much greater 15 is than 8 (also additive). For the third, the unknown in the box is 12 times as large as 42 (multiplicative). In the fourth, 14 is 3 times the size of the unknown in the box (multiplicative). In the fifth, the box tells me how many copies of 5 are in 14 (multiplicative). It may be necessary to nudge students for a response that tells how the box works in the comparison.***

D. Content: Fractions (Meaning)



If Luis painted ⅓ as many pictures as Carlos, what fraction of all the paintings were painted by Carlos?

> ***This question opens up the chance to talk about fractions. If the number of paintings Luis painted is ⅓ of the number of paintings Carlos painted, or ⅓, we could be thinking of 4 parts.***

$\frac{1}{3}$ of a number is 18	$\frac{1}{3}$ of 18

How could you use this diagram to show that Luis painted ⅓ as many pictures as Carlos? What would each region represent?

> ***If the previous question was too abstract, this question may prompt a conversation about fractions, additively and multiplicatively at the same time, and the meaning of the whole. Each section could represent the number of pictures Luis painted. So, three sections represent the number of pictures Carlos painted. All four sections represent the number of pictures painted by Luis and Carlos combined.***

How are these two statements alike? How are they different?

> ***Though similar in appearance, these statements have different meanings. The first statement means that 18 is ⅓ of some number, or that some number is 3 times 18. The second statement refers to a number that is ⅓ of 18, or that 18 is 3 times that number. Can students see the difference and make sense of each?***

E. Extension: Comparison (Multiplicative)

If Carlos always paints 3 times as many pictures as Luis, what will happen if the number of pictures Luis paints increases by 2?

> ***The number of pictures Carlos paints will increase by 3 times as many, or by 6.***

4.1 Shareables*

Use the information to complete the task.

Carlos painted 18 pictures. He painted 3 times as many pictures as Luis painted.

Make an equation to show the number of pictures Luis painted. Move numbers and symbols to the lines. Use the blank box to represent the number of pictures Luis painted.

__________ = ____

□ 3 18 + − × ÷

A.

3 times a number is 729

3 times 729 is a number

C.

$13 - \square = 8$

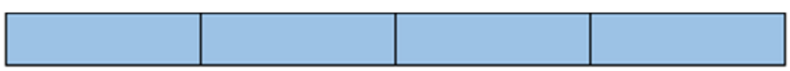
$15 - 8 = \square$

$12 \times 42 = \square$

$14 \div \square = 3$

$14 \div 5 = \square$

D.



$\frac{1}{3}$ of a number is 18	$\frac{1}{3}$ of 18

4.2 Place Value

This activity focuses on student thinking about place value.

Place Value: Knowledge of place value is essential, and students should develop ways of thinking about place value that enable them to see the relationships between places. For example, they can think of a value in one place as 10 times that same value in the place to the right (or a *bundle of 10*), and they can carry that thinking between any places in any direction. They should be able to use that understanding effortlessly to compose and decompose quantities and to connect place-value understanding to operations.

ITEM ALIGNMENT

CCSS: 4.NBT.A.2

This item focuses on understanding whole-number place value. However, it also provides opportunities to talk about larger place-value concepts and relationships, as well as decimal place value.

THE CONVERSATION STARTER

Complete the comparisons. Move the correct symbol to each box.

$5,362$ ☐ $500 + 300 + 60 + 2$

$5,362$ ☐ 5 thousands + 36 hundreds + 2 tens

$5,362$ ☐ 5 thousands + 3 hundreds + 2 tens + 6 ones

$<$ $>$ $=$

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Content: Place Value (Meaning)

How does place value work?

Someone tells you that 2,001 = 21 because zeros have no value. What do you say?

How does 200 compare to 2,000? Which comparison connects to place value?

What is the relationship between the 2s in each number below? Can you describe the relationship in two ways?

5,221 2,521

B. Content: Place Value (Comparison)

We sometimes use the terms *greater than* or *less than*. What makes one number greater than another number?

Which number is larger?

634 123

C. Content: Place Value (Meaning)

How many tens are in 5,362?

How many tens are in 60?

How many tens are in 300?

- What about in 310?

How many ones are in 300?

- What about in 310?
- Is it correct to say 5,362 is “53 hundreds + 6 tens + 2 ones”?

Is it correct to say “5,362 is 536 tens + 2 ones”?

D. Content: Place Value (Number Line)

Look at this number line. What is the relationship between one of the larger segments and one of the smaller segments?

Does changing what number is represented by the mark farthest to the right on the number line change the number represented by a point placed on the number line? Does it change the relationship between the smaller and larger marks?

E. Content: Place Value (Meaning)

I want to add 5,362 + 78. Can I start by adding 5 and 7? Why or why not?

5,362 78

F. Extension: Place Value (Decimals)

The number 5,362 can be written in expanded form like this: 5,362 = 5 × 1,000 + 3 × 100 + 6 × 10 + 2. How could you show 536.2 this way?

CONVERSATION PATHS (ANNOTATED)*

A. Content: Place Value (Meaning)

How does place value work?

> ***A value in one place is 10 times that same value in the place to its right (or a bundle).***

Someone tells you that 2,001 = 21 because zeros have no value. What do you say?

> ***The value of a digit in a number is based on its place in the number. In both 2,001 and 21, there is 1 one. The zeros in 2,001 make it look like there are no tens and no hundreds, but actually there are 200 tens or 20 hundreds. The number 21 has 2 tens.***

How does 200 compare to 2,000? Which comparison connects to place value?

> ***2,000 is 1,800 more than 200. It is also 10 times as great as 200.***

What is the relationship between the 2s in each number below? Can you describe the relationship in two ways?

5,221 2,521

> ***5,221: The 2 in the hundreds place is 10 times as great as the 2 in the tens place. The 2 in the tens place is one-tenth as great as the 2 in the hundreds place.***
>
> ***2,521: The 2 in the thousands place is 100 times as great as the 2 in the tens place. The 2 in the tens place is one-hundredth as great as the 2 in the thousands place.***

B. Content: Place Value (Comparison)

We sometimes use the terms *greater than* or *less than*. What makes one number greater than another number?

> ***Most students will assume that a number greater than another number is a larger quantity, but it depends on the attribute being compared.***

Which number is larger?

634 **123**

> ***The number 123 is taller, but the number 634 represents a larger quantity.***

C. Content: Place Value (Meaning)

How many tens are in 5,362?

> ***Continue if students answer anything other than 536. Note that students sometimes say "6" if they mistake the question as "What digit is in the tens place?"***

How many tens are in 60?

> ***6***

How many tens are in 300?

> ***30***

- What about in 310?

> ***31***

How many ones are in 300?

> ***300***

- What about in 310?

 310

- Is it correct to say 5,362 is “53 hundreds + 6 tens + 2 ones?”

 Yes.

Is it correct to say “5,362 is 536 tens + 2 ones”?

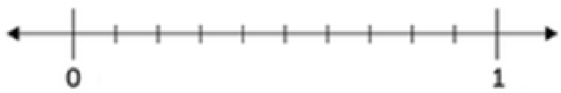
 Yes.

D. Content: Place Value (Number Line)

Look at this number line. What is the relationship between one of the larger segments and one of the smaller segments?

There are 10 smaller segments between each larger segment; each larger segment is equal to a bundle of 10 smaller segments.

Does changing what number is represented by the mark farthest to the right on the number line change the number represented by a point placed on the number line? Does it change the relationship between the smaller and larger marks?



Yes, changing the number that the mark farthest to the right represents changes the number represented by a point on the number line, because it changes the scale of the number line. It doesn’t change the relationship between the smaller and larger segment, though, because each larger segment is always equal to 1 bundle of 10 smaller segments.

E. Content: Place Value (Meaning)

I want to add 5,362 + 78. Can I start by adding 5 and 7? Why or why not?

5,362 78

It wouldn’t make sense because the 5 in 5,362 represents 5 thousands and the 7 in 78 represents 7 tens. If you add the 5 and 7, you have to understand their place value: 5 + 7 = 12, but 5,000 + 70 = 5,070.

F. Extension: Place Value (Decimals)

The number 5,362 can be written in expanded form like this: 5,362 = 5 × 1,000 + 3 × 100 + 6 × 10 + 2. How could you show 536.2 this way?

5 × 100 + 3 × 10 + 6 × 1 + 2 x 1⁄10

4.2 Shareables*

Complete the comparisons. Move the correct symbol to each box.

5, 362 ☐ 500 + 300 + 60 + 2

5, 362 ☐ 5 thousands + 36 hundreds + 2 tens

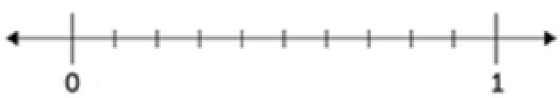
5, 362 ☐ 5 thousands + 3 hundreds + 2 tens + 6 ones

$<$ $>$ $=$

A. 5,221 2,521

B. 634 123

D.



5,362 78

4.3 Fractions

This activity focuses on student thinking about the relationship between fractions and decimals.

Fractions: A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

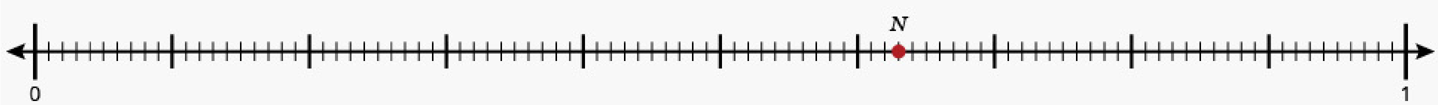
ITEM ALIGNMENT

CCSS: 4.NF.C.6

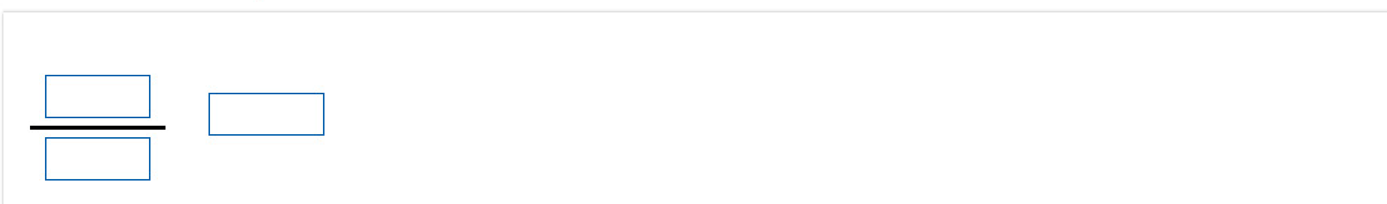
This item focuses on the relationship between fractions and decimals. However, it also provides opportunities to talk about number lines, place value, multiplicative comparison, fraction concepts, and fractions and decimals as numbers.

THE CONVERSATION STARTER

Use the number line to answer the question.



What number is located at point N on the number line? Write the number as a fraction and as a decimal. Enter the answers in the boxes.



CONVERSATION PATHS (QUESTIONS ONLY)*

A. Content: Fractions (Meaning)

What can you tell me about the fraction $\frac{72}{88}$?

Is $\frac{72}{88}$ one number or two numbers?

What do the numbers in $\frac{72}{88}$ mean? How do they relate to one another?

How can you compare $\frac{72}{88}$ to $\frac{1}{88}$?

$$72/88 \qquad 1/88$$

$$\frac{72}{88} \qquad \frac{1}{88}$$

- How can you compare $\frac{72}{88}$ to $\frac{72}{44}$?

B. Content: Fractions (Decimals, Comparison)

What does 0.4 mean?

- How does the way we name “four-tenths” connect to fractions?
- What does 0.43 mean?
- Which is greater, 0.43 or 0.5? Why? How would we write them as fractions?

C. Content: Fractions (Equality, Number Line)

In the original question, what does the number line show? What else?

- Using the number line, can you explain why $\frac{4}{10}$ is equivalent to $\frac{40}{100}$?
- Can you show how $\frac{2}{10}$ is equal to $\frac{1}{5}$?
- What are some other ways to correctly write the number located at point *N*?

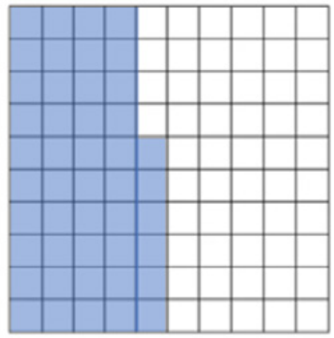
D. Content: Fractions (Comparison, Decimals)

How do these numbers compare to one another?

- 0.6 and 6
- $\frac{6}{100}$ and 6

E. Content: Fractions (Representation, Decimals)

Look at the grid.



- Tell me how the grid can represent 0.46.
- Tell me how the grid can represent $4\frac{6}{10}$.
- Tell me how the grid can represent $\frac{46}{10}$.

F. Content: Fractions (Meaning)

Look at the number line in the original question. Is there a number between $\frac{65}{100}$ and $\frac{64}{100}$?

Look at the number line in the original question. If you change the 1 to a 10, what does point *N* represent?

- How does this new value of *N* compare to the original value of *N*?
- What happens to the value of *N* if we change the 1 to a 5 on the number line?

CONVERSATION PATHS (ANNOTATED)*

A. Content: Fractions (Meaning)

What can you tell me about the fraction $\frac{72}{88}$?

Answers will vary. The goal of these questions is to determine the level of students' understanding of the idea of a fraction. Do they understand $\frac{72}{88}$ as simply 72 out of 88, or as 72 parts of a whole, 72 parts of a set, or 72 parts of a line divided into 88 equal parts? Do they understand the relationship to the unit fraction $\frac{1}{88}$? The fraction $\frac{72}{88}$ is 72 copies of $\frac{1}{88}$. Do they see it as 72 divided by 88 or as a comparison of 72 to 88?

Is $\frac{72}{88}$ one number or two numbers?

A fraction looks like two numbers, but it is a single number.

What do the numbers in $\frac{72}{88}$ mean? How do they relate to one another?

72 is the numerator. 88 is the denominator. There are 72 out of 88 parts of a whole. The goal of this question is to determine how students understand the idea of a fraction.

How can you compare $\frac{72}{88}$ to $\frac{1}{88}$?

$$72/88 \qquad 1/88$$

$$\frac{72}{88} \qquad \frac{1}{88}$$

Examples: $\frac{72}{88}$ is 72 times as much as $\frac{1}{88}$. It is also true that $\frac{72}{88}$ is $\frac{71}{88}$ larger than $\frac{1}{88}$. It's also true that $\frac{72}{88}>\frac{1}{88}$.

- How can you compare $\frac{72}{88}$ to $\frac{72}{44}$?

  Multiplicative examples include $\frac{72}{88}$ is one-half times as much as $\frac{72}{44}$, and that $\frac{72}{44}$ is 2 times as much as $\frac{72}{88}$.

B. Content: Fractions (Decimals, Comparison)

What does 0.4 mean?

0.4 is 4 parts of a whole that is divided into 10 equal parts. Or, 0.4 is 4 copies of 0.1, where 0.1—or one-tenth—is what we see when a whole is cut up into 10 equal parts.

- How does the way we name "four-tenths" connect to fractions?

  We say "four-tenths," which indicates how to represent it as a fraction: $\frac{4}{10}$.

- What does 0.43 mean?

  0.43 is 43 parts of a whole that is divided into 100 equal parts. Or, 0.43 is 43 copies of 0.01, where 0.01—or one-hundredth—is one part when a whole is divided into 100 equal parts.

- Which is greater, 0.43 or 0.5? Why? How would we write them as fractions?

  0.5 is greater than 0.43. The number 0.5 represents 5 parts of a whole divided into 10 equal parts, or 50 parts of a whole divided into 100 equal parts. It can be written as $\frac{5}{10}$ or $\frac{50}{100}$. The number 0.43 represents 43 parts of a whole divided into 100 equal parts. It can be written as $\frac{43}{100}$. 50 parts of 100 is greater than 43 parts of 100. Or, 0.43 is 43 copies of $\frac{1}{100}$, and 0.50 is 50 copies of $\frac{1}{100}$. Since 50 copies of $\frac{1}{100}$ is greater than 43 copies of $\frac{1}{100}$, 0.5 is greater than 0.43.

C. Content: Fractions (Equality, Number Line)

In the original question, what does the number line show? What else?

> ***A number line showing 0 to 1, and each section is divided into 10 segments, and each segment is divided into 10 smaller segments.***

- Using the number line, can you explain why $\frac{4}{10}$ is equivalent to $\frac{40}{100}$?

  The number line is divided into 10 large segments. Each large segment represents $\frac{1}{10}$ of the whole number line. The distance to the fourth large segment represents $\frac{4}{10}$. The number line is also divided into 100 small segments. Each large segment is composed of 10 small segments. So, the distance to the fourth large segment also represents $\frac{40}{100}$.

- Can you show how $\frac{2}{10}$ is equal to $\frac{1}{5}$?

  The number line is divided into 10 large segments; each large segment is $\frac{1}{10}$ of the number line. If 2 of those segments are counted as a pair, there are 5 of those pairs. So, every $\frac{2}{10}$ is equal to $\frac{1}{5}$.

- What are some other ways to correctly write the number located at point *N*?

  $\frac{72}{100}$; $\frac{36}{50}$; 0.72; 0.7200, etc.

D. Content: Fractions (Comparison, Decimals)

How do these numbers compare to one another?

- 0.6 and 6

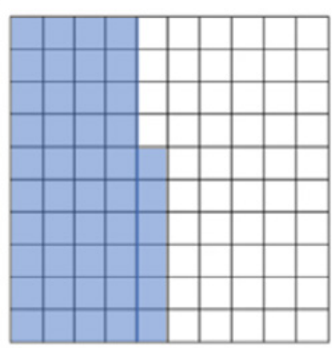
  Multiplicatively, 0.6 is $\frac{1}{10}$ the size of 6, or 6 is 10 times the size of 0.6.

- $\frac{6}{100}$ and 6

  Multiplicatively, $\frac{6}{100}$ is $\frac{1}{100}$ the size of 6, or 6 is 100 times as great as $\frac{6}{100}$.

E. Content: Fractions (Representation, Decimals)

Look at the grid.



- Tell me how the grid can represent 0.46.

  When the entire grid represents 1, each column/row represents 0.1, and each box represents 0.01.

- Tell me how the grid can represent $4\frac{6}{10}$.

  When the entire grid represents 10, each column/row represents 1, and each box represents 0.1.

- Tell me how the grid can represent $\frac{46}{10}$.

  When the entire grid represents 10, each column/row represents 1, and each box represents 0.1.

F. Content: Fractions (Meaning)

Look at the number line in the original question. Is there a number between $\frac{65}{100}$ and $\frac{64}{100}$?

> ***This question is about the meaning of the number line. While the number line does not have a mark for such a number, that number does exist. The space between marks on the number line can be divided into smaller and smaller increments. Can students come up with something like $\frac{645}{1000}$ on their own?***

Look at the number line in the original question. If you change the 1 to a 10, what does point *N* represent?

> ***6.3 or $6\frac{3}{10}$***

- How does this new value of *N* compare to the original value of *N*?

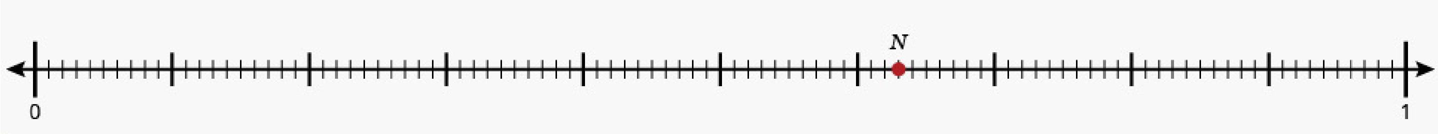
  It is 10 times the original value, because we multiplied by 10.

- What happens to the value of *N* if we change the 1 to a 5 on the number line?

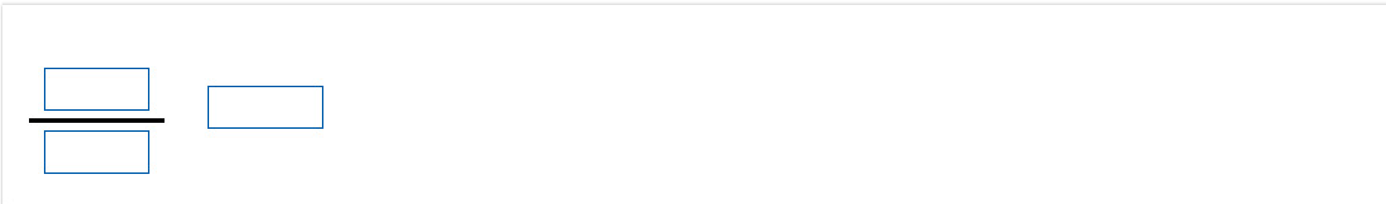
  It will be 5 times the original value, because we multiplied by 5. Also, the value of* N *will be half of what it became when the 1 was changed to a 10, because 5 is half of 10. So, the value of* N *will be 3.15, or $3\frac{15}{100}$.

4.3 Shareables*

Use the number line to answer the question.



What number is located at point N on the number line? Write the number as a fraction and as a decimal. Enter the answers in the boxes.

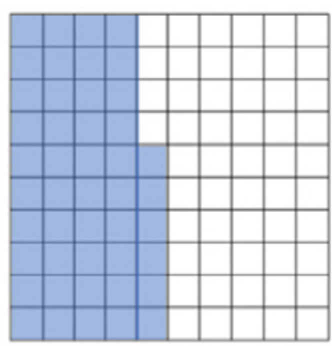


A.

72/88 1/88

$\frac{72}{88}$ $\frac{1}{88}$

E.



4.4 Fractions

This activity focuses on student thinking about ordering fractions.

Fractions: A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

ITEM ALIGNMENT

CCSS: 4.NF.A.2

This item focuses on ordering fractions. However, it also provides opportunities to talk about the meanings of the numerator and denominator, the importance of the same whole, comparisons, and flexibility with fraction understandings.

THE CONVERSATION STARTER

Move the fractions so that they are in order from least to greatest.

least ☐ ☐ ☐ ☐ greatest

$\frac{3}{8}$ $\frac{5}{6}$ $\frac{11}{12}$ $\frac{1}{2}$

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Content: Fractions (Meaning)

What is a fraction?

Is a fraction two numbers or one number?

What other words are used when talking about fractions? What do they mean?

What do you think of when you see the fraction $\frac{3}{8}$?

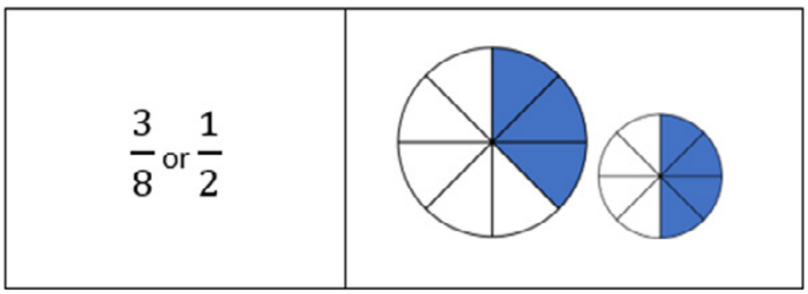
- If it means "3 out of 8," then what does $\frac{8}{3}$ mean?

Can you write $\frac{3}{8}$ as a sum of fractions?

- Can you write $\frac{3}{8}$ as a product of fractions?

B. Content: Fractions (Comparison)

Which is bigger, $\frac{3}{8}$ or $\frac{1}{2}$?



In the original problem, what is meant by "least" and "greatest"?

How much bigger is $\frac{6}{8}$ than $\frac{3}{8}$?

- How much bigger is $\frac{3}{8}$ than $\frac{3}{16}$?
- How much bigger is $\frac{3}{8}$ than $\frac{6}{8}$?

How are $\frac{3}{8}$ and $\frac{5}{8}$ alike?

- How are they different?
- Which is greater, and why?

How are $\frac{3}{8}$ and $\frac{3}{5}$ alike?

- How are they different?
- Which is greater, and why?

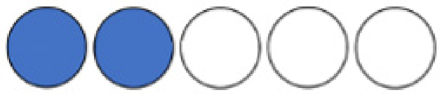
C. Content: Fractions (Comparison)

In the original problem, which number is closest to 1? How do you know?

- Which number is closest to zero? How do you know?

D. Content: Fractions (Meaning)

Look at this diagram.



How could you use this diagram to show that $\frac{2}{5}$ is 2 times as large as $\frac{1}{5}$?

How could you use this diagram to show that $\frac{2}{5}$ is 2 times as large as $\frac{2}{10}$?

How could the diagram represent $\frac{2}{5}$?

There are five circles in the diagram, but is there a way you could use the five circles to show $\frac{2}{3}$?

There are five circles in the diagram, but is there a way you could use the five circles to show $\frac{3}{2}$?

E. Content: Fractions (Meaning, Number Line)

Look at this number line.

How could you use this number line to explain the following statements?

- $\frac{2}{5}$ is 2 times the value of $\frac{1}{5}$
- $\frac{2}{5}$ is 2 times the value of $\frac{2}{10}$

CONVERSATION PATHS (ANNOTATED)*

A. Content: Fractions (Meaning)

What is a fraction?

> ***A fraction is a number, just as 3, 12, or 0 are numbers. It represents a point on the number line.***

Is a fraction two numbers or one number?

> ***A fraction looks like two numbers, but it is a single number.***

What other words are used when talking about fractions? What do they mean?

> ***Numerator and denominator, for example. The denominator tells the number of partitions of the whole, the numerator tells how many copies of that partition we have.***

What do you think of when you see the fraction 3/8?

> ***Do your students first think of fractions as numbers on a number line? Do they see them as pieces of a pie? As parts of a set? As fraction bars?***

- If it means "3 out of 8," then what does 8/3 mean?

Can you write 3/8 as a sum of fractions?

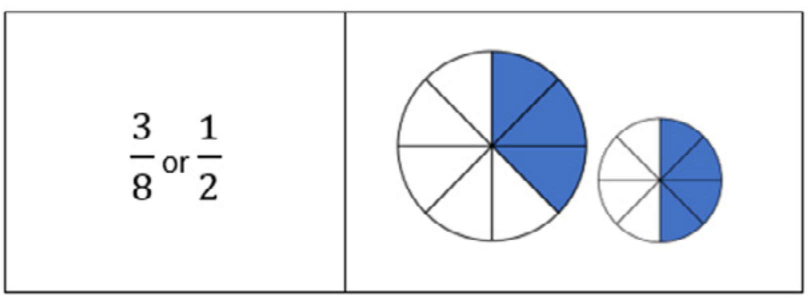
> ***Yes. For example, 1/8 + 1/8 + 1/8.***

- Can you write 3/8 as a product of fractions?

  Fractions may be viewed additively or multiplicatively. Ideally, students have agility representing fractions in both ways. For example, 3/8 can be written as 2/8 + 1/8, and students may also see 3/8 as 3 copies of 1/8, or 3 x 1/8.

B. Content: Fractions (Comparison)

Which is bigger 3/8 or 1/2?



> ***These are tandem questions to elicit dissonance. First show the fractions, then show the circle diagrams. The use of "bigger" here is intentionally ambiguous to drive conversation. Do we mean bigger in value on the number line or physically bigger? If it means bigger in value on a number line, then students have to match the wholes.***

In the original problem, what is meant by "least" and "greatest"?

> ***"Least" refers to farthest left on the number line (or simply the closest to zero for positive numbers), and "greatest" refers to farthest right (or farthest from zero) on the number line.***

How much bigger is 6/8 than 3/8?

- How much bigger is 3/8 than 3/16?
- How much bigger is 3/8 than 6/8?

To further leverage the ambiguity around "bigger," the thinking can be additive or multiplicative. Do students see the multiplicative approach? Do they see that $\frac{6}{8}$ is 2 times as much as $\frac{3}{8}$?)

- ***Do they see that $\frac{3}{8}$ is 2 times as much as $\frac{3}{16}$?***
- ***$\frac{3}{8}$ is not bigger than $\frac{6}{8}$. It is half as big.***

How are $\frac{3}{8}$ and $\frac{5}{8}$ alike?

- How are they different?
- Which is greater, and why?

Both are numbers. Both are fractions with a denominator of 8, which means they both represent a certain number of one-eighths.)

- ***They have different numerators, which means they represent a different number of one-eighths.***
- ***If the wholes are the same size, the number $\frac{5}{8}$ is greater than the number $\frac{3}{8}$ because 5 one-eighths are greater than 3 one-eighths.***

How are $\frac{3}{8}$ and $\frac{3}{5}$ alike?

- How are they different?
- Which is greater, and why?

Both are numbers. Both are fractions with a numerator of 3, which means they both represent 3 parts of something divided into equal parts.

- ***They have different denominators, which means they are divided into a different number of equal parts.***
- ***If the wholes are the same size, the number $\frac{3}{5}$ is greater than the number $\frac{3}{8}$ because the more pieces you divided a whole into, the smaller each piece is. So, 3 one-fifths is greater than 3 one-eighths.***

C. Content: Fractions (Comparison)

In the original problem, which number is closest to 1? How do you know?

In the numbers $\frac{5}{6}$ and $\frac{11}{12}$, the numerator (the number of parts) is only 1 less than the denominator (the number of equal parts the whole is divided into). The number $\frac{11}{12}$ is closer to 1 than the number $\frac{5}{6}$ because twelfths are smaller than sixths, so $\frac{1}{12}$ is a smaller distance from 1 than $\frac{1}{6}$ is.

- Which number is closest to zero? How do you know?

The number $\frac{3}{8}$ is closest to 0 because it is less than $\frac{1}{2}$, and all the other numbers are either $\frac{1}{2}$ or greater than $\frac{1}{2}$.

D. Content: Fractions (Meaning)

Look at this diagram.

How could you use this diagram to show that $\frac{2}{5}$ is 2 times as large as $\frac{1}{5}$?

Since there are 5 circles, one shaded circle is $\frac{1}{5}$. Two shaded circles are $\frac{2}{5}$, which is 2 times as large as $\frac{1}{5}$.

How could you use this diagram to show that $\frac{2}{5}$ is 2 times as large as $\frac{2}{10}$?

If we divide each circle in half, each circle can be thought of as $\frac{1}{5}$ or $\frac{2}{10}$. $\frac{2}{5}$ is 2 shaded whole-circles, which is 2 times as large as $\frac{2}{10}$, which is only 1 shaded whole-circle.

How could the diagram represent $\frac{2}{5}$?

> ***If the whole is the collection of 5 circles, then a single circle is $\frac{1}{5}$ of the collection. The shaded circles represent 2 copies of $\frac{1}{5}$, or $\frac{2}{5}$.***

There are five circles in the diagram, but is there a way you could use the five circles to show $\frac{2}{3}$?

> ***If the whole is a collection of 3 unshaded circles, then a single circle is $\frac{1}{3}$ of the whole. The shaded circles represent 2 copies of $\frac{1}{3}$, or $\frac{2}{3}$.***

There are five circles in the diagram, but is there a way you could use the five circles to show $\frac{3}{2}$?

> ***If the whole is the collection of 2 unshaded circles only, then a single circle is $\frac{1}{2}$ of the collection. The unshaded circles represent 3 copies of $\frac{1}{2}$, or $\frac{3}{2}$.***
>
> ***Note: This cluster of items was inspired by Thompson, P. W., & Saldanha, L. A. (2003). Fractions and multiplicative reasoning. Research companion to the principles and standards for school mathematics, 95-113.***

E. Content: Fractions (Meaning, Number Line)

Look at this number line.

How could you use this number line to explain the following statements?

- $\frac{2}{5}$ is 2 times the value of $\frac{1}{5}$

  For example, think of the number line as from 0 to 1. There are 5 equal segments between 0 and 1, so each small segment represents $\frac{1}{5}$. Two small segments are 2 times as long as one segment, so $\frac{2}{5}$ is 2 times the value of $\frac{1}{5}$.

- $\frac{2}{5}$ is 2 times the value of $\frac{2}{10}$

  For example, think of the number line as from 0 to 1. There are 5 equal segments between 0 and 1, so each small segment represents $\frac{1}{5}$. You could divide each of the 5 small segments in half to get segments that represent $\frac{1}{10}$. Every $\frac{1}{5}$ segment is equal to 2 one-tenth segments, so $\frac{2}{5}$ is 2 times the value of $\frac{2}{10}$.

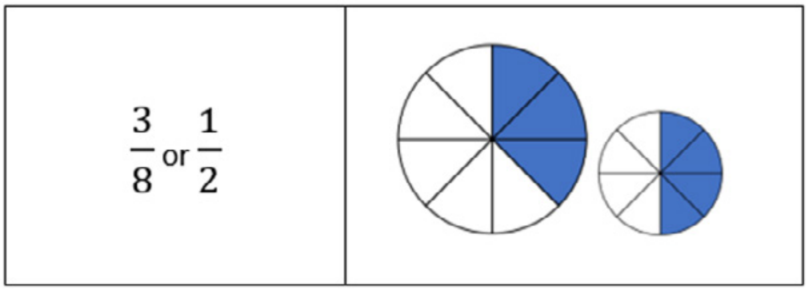
4.4 Shareables*

Move the fractions so that they are in order from least to greatest.

least ☐ ☐ ☐ ☐ greatest

$\frac{3}{8}$ $\frac{5}{6}$ $\frac{11}{12}$ $\frac{1}{2}$

B.



D.

E.

4.5 Fractions

This activity focuses on student thinking about multiplying fractions and whole numbers.

Fractions: A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

ITEM ALIGNMENT

CCSS: 4.NF.B.4b

This item focuses on multiplying fractions and whole numbers. However, it also provides opportunities to talk about meanings of fractions, operations, and comparisons.

THE CONVERSATION STARTER

Are the expressions in the table equal to $4 \times \frac{2}{6}$? Choose "Yes" or "No" for each expression.

Expression	Equal to $4 \times \frac{2}{6}$?
$4 \times \frac{4}{6}$	Yes / No
$4 \times \frac{1}{3}$	Yes / No
$8 \times \frac{1}{6}$	Yes / No

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Content: Fractions (Comparison)

Are there differences between $4\frac{2}{6}$, $4 + \frac{2}{6}$, and $4 \times \frac{2}{6}$? If so, what are they?

- How are $4 \times \frac{2}{6}$ and $2 \times \frac{4}{6}$ alike? How are they different?
- How are 4 and $\frac{4}{6}$ alike? How are they different?

B. Content: Fractions (Equality)

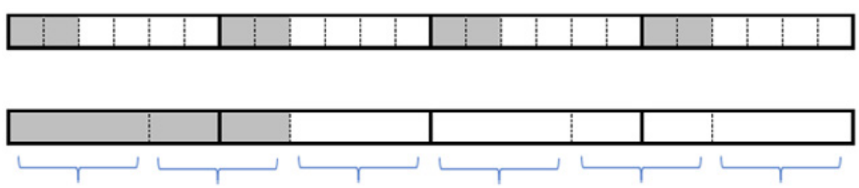
What are some other ways to write $\frac{4}{6}$?

- What is another way to write $4 \times \frac{1}{6}$ using addition?
- How could knowing how to write $\frac{4}{6}$ as a multiplication expression help you find $4 \times \frac{4}{6}$?

C. Content: Comparison (Fractions, Multiplicative)

If $4 \times \frac{2}{6}$ means *4 copies of* $\frac{2}{6}$, what does $\frac{2}{6} \times 4$ mean?

- How are $4 \times \frac{2}{6}$ and $\frac{2}{6} \times 4$ alike? How are they different?
- Take a moment to look at these two diagrams.



  - Tell me when you see $4 \times \frac{2}{6}$ in the diagram.
  - Tell me when you see $\frac{2}{6} \times 4$ in one of the diagrams.
  - Could these models show that $4 \times \frac{2}{6}$ and $\frac{2}{6} \times 4$ have the same value?
- Does it seem odd that $4 \times \frac{2}{6}$ and $\frac{2}{6} \times 4$ have the same value?

D. Content: Fractions (Multiplication, Representation)

What question could you write that could be represented by $4 \times \frac{2}{6}$?

- What problem could you write that could be represented by $\frac{4}{6} \times 4$?

E. Content: Fractions (Multiplication, Meaning)

When you multiply two numbers, is the answer always greater than both numbers? Why or why not?

- Without calculating, is the product of $\frac{4}{5} \times 3$ greater than or less than 3? How do you know?
- Without calculating, is the product of $\frac{8}{6} \times 3$ greater than or less than 3? How do you know?
- Without calculating, how does $4 \times \frac{1}{3}$ compare to $8 \times \frac{1}{6}$?

CONVERSATION PATHS (ANNOTATED)*

A. Content: Fractions (Comparison)

Are there differences between $4\frac{2}{6}$, $4 + \frac{2}{6}$, and $4 \times \frac{2}{6}$? If so, what are they?

> ***$4\frac{2}{6}$ and $4 + \frac{2}{6}$ are the same because $4 + \frac{2}{6}$ means the numbers are to be added, which is the same as $4\frac{2}{6}$. $4 \times \frac{2}{6}$ is different because it represents finding 4 sets of $\frac{2}{6}$, which is $\frac{8}{6}$, or $1\frac{2}{6}$.***

- How are $4 \times \frac{2}{6}$ and $2 \times \frac{4}{6}$ alike? How are they different?

  The first expression means 4 copies of $\frac{2}{6}$, or 8 copies of $\frac{1}{6}$. The second expression means 2 copies of $\frac{4}{6}$, or 8 copies of $\frac{1}{6}$. So they start with different meanings but end up representing the same value.

- How are 4 and $\frac{4}{6}$ alike? How are they different?

  Both are numbers. 4 is a whole number; $\frac{4}{6}$ is a fraction. In the whole number, 4 represents 4 wholes. In the fraction, 4 represents 4 parts of a whole that is partitioned into 6 equal parts.

B. Content: Fraction (Equality)

What are some other ways to write $\frac{4}{6}$?

> ***For example, $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$, or $\frac{2}{6} + \frac{2}{6}$; $\frac{1}{6} + \frac{3}{6}$, or 4 x $\frac{1}{6}$, or $\frac{2}{3}$.***

- What is another way to write $4 \times \frac{1}{6}$ using addition?

  $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

- How could knowing how to write 4/6 as a multiplication expression help you find $4 \times \frac{1}{6}$?

  Think of it as 4 copies of 4 copies of $\frac{1}{6}$; the fraction $\frac{4}{6} = 4 \times \frac{1}{6}$, so $4 \times \frac{4}{6}$ can be written as $4 \times 4 \times \frac{1}{6} = \frac{16}{1} \times \frac{1}{6} = \frac{16}{6}$.

C. Content: Comparison (Fractions, Multiplicative)

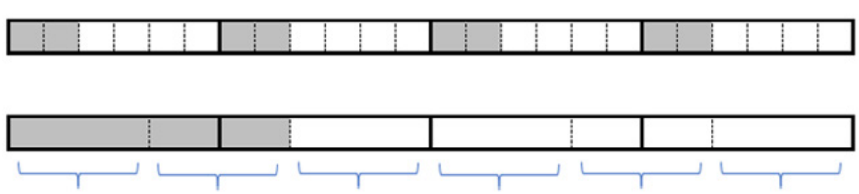
If $4 \times \frac{2}{6}$ means 4 copies of $\frac{2}{6}$, what does $\frac{2}{6} \times 4$ mean?

> ***It could mean $\frac{2}{6}$ copies of 4, which more specifically would mean 2 copies of $\frac{1}{6}$ of 4.***

- How are $4 \times \frac{2}{6}$ and $\frac{2}{6} \times 4$ alike? How are they different?

  Both expressions are equal to $\frac{8}{6}$, but the meaning of each expression is different. $4 \times \frac{2}{6}$ means 4 copies of $\frac{2}{6}$, and $\frac{2}{6} \times 4$ means 2 copies of $\frac{1}{6}$ of 4.

- Take a moment to look at these two diagrams.



  - Tell me when you see $4 \times \frac{2}{6}$ in the diagram.

    In the top model, if each square represents $\frac{1}{6}$ then each shaded area represents $\frac{2}{6}$. There are 4 sets of $\frac{6}{6}$, so there are 4 wholes. The total of the shaded regions is $4 \times \frac{2}{6}$.

  - Tell me when you see $\frac{2}{6} \times 4$ in one of the diagrams.

    In the bottom model, the total region is divided into fourths (the solid dividers) and sixths (the groupings shown underneath). We can consider each of the fourths to represent one whole, so the diagram shows 4 wholes in all. We can see that $\frac{2}{6}$ of that is shaded. So this diagram can represent $\frac{2}{6}$ of a copy of 4, or $\frac{2}{6} \times 4$.

- Could these models show that $4 \times \frac{2}{6}$ and $\frac{2}{6} \times 4$ have the same value?

  If we imagine moving the shaded regions in the top diagram to the left, we can see that the shaded regions of both are the same.

- Does it seem odd that $4 \times \frac{2}{6}$ and $\frac{2}{6} \times 4$ have the same value?

  When you really think about it without assuming the commutative property for multiplication, it's entirely reasonable to wonder why this should be true.

D. Content: Fractions (Multiplication, Representation)

What question could you write that could be represented by $4 \times \frac{2}{6}$?

For example: Shane works on his project for $\frac{2}{6}$ of an hour every day for 4 days. How long does he work on his project over the 4 days?

- What problem could you write that could be represented by $\frac{4}{6} \times 4$?

  For example: Surita spent 4 hours on a project. She spent $\frac{4}{6}$ of the time painting. How much time did Surita spend painting?

E. Content: Fractions (Multiplication, Meaning)

When you multiply two numbers, is the answer always greater than both numbers? Why or why not?

No. It depends on the value of the factors.

- Without calculating, is the product of $\frac{4}{5} \times 3$ greater than or less than 3? How do you know?

  The product will be less than 3 because $\frac{4}{5} \times 3$ means that 3 is partitioned into 5 parts but only 4 of those parts are included.

- Without calculating, is the product of $\frac{8}{6} \times 3$ greater than or less than 3? How do you know?

  The product will be greater than 3 because $\frac{8}{6} \times 3$ is 8 parts of 3 partitioned into 6 parts.

- Without calculating, how does $4 \text{ x } \frac{1}{3}$ compare to $8 \text{ x } \frac{1}{6}$?

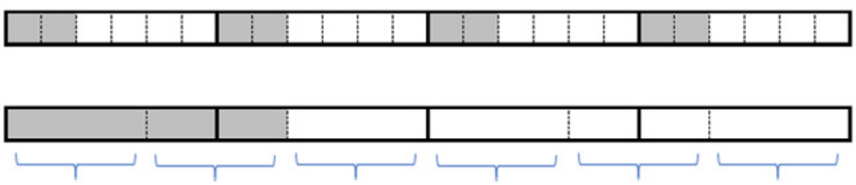
  They are equal. Four is half of 8, but $\frac{1}{3}$ is 2 times as great as $\frac{1}{6}$.

4.5 Shareables*

Are the expressions in the table equal to $4 \times \frac{2}{6}$? Choose "Yes" or "No" for each expression.

Expression	Equal to $4 \times \frac{2}{6}$?
$4 \times \frac{4}{6}$	Yes / No
$4 \times \frac{1}{3}$	Yes / No
$8 \times \frac{1}{6}$	Yes / No

C.



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